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THERMODYNAMIC ANALYSIS AND OPTIMIZATION BASED ON EXERGY FLOW FOR A TWO-STAGED PULSE TUBE REFRIGERATOR

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ABSTRACT

A control thermodynamic model for a two-stage Pulse Tube Refrigerators (PTRs) is developed based on exergy flow in the refrigerator. The model includes flow conductance, heat transfer effectiveness, and conduction heat transfer parameters for regenerators in both stages. It is assumed that a phase shift controller exists in both stages to control the phase shift between the pressure and mass flow rates that can control and optimize the performance of the refrigerator. The effects of the allocation of the values of flow conductance and ineffectiveness parameters in the regenerators, the mid-stage temperature, and the phase shift in each stage on the performance of the refrigerator are investigated. Important dimensionless parameters controlling the thermodynamic performance of two-stage PTRs, including a model to quantify the magnitude of different components of irreversibility in the regenerators, is developed and discussed.

KEYWORDS: Exergy analysis, Multi-stage, Cryocoolers, Pulse tubes, Irreversibility

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INTRODUCTION

Multi-stage Pulse Tube Refrigerators (MPTRs) are attractive cooling systems for a variety of applications and their staging configurations provide the flexibility to conveniently change the various stage temperatures for different system applications [1-4]. Recently, interest in the design and development of high frequency multistage PTRs has been initiated and low, no-load temperatures have been obtained [5]. Numerical calculation to evaluate the performance of multi-stage PTRs has also been reported [6].

In general, the model of control thermodynamics (finite time thermodynamics) can be applied for analysis and optimization of multi-stage refrigerators [7, 8]. Different thermodynamic models to analyze and optimize multi-stage refrigerators exist in the literature and use different models to calculate the irreversibility for internal processes and external heat transfer with reservoirs [10-14], to just name a few. Thermodynamic optimization of two-stage PTRs has been carried out assuming that the only irreversibility in the regenerator is due to conduction heat transfer [15]. In this study, we assume that a general phase-shifter exists to control the phase shift between the mass flow rate and pressure in the pulse tube for each stage. In addition, we include a more realistic model of the regenerator in both stages to include the effects of the void, pressure drop, heat transfer ineffectiveness, and conduction heat transfer in the regenerator. At least a first order thermodynamic model for the regenerator is necessary to be able to provide a more realistic thermodynamic model to account for its contribution to the irreversibility of two-stage PTRs. In this study an exergy-based analysis for two-stage PTRs is performed to quantify the losses and optimize the refrigerator [16-18]. Important dimensionless parameters controlling the performance of the refrigerators are obtained and discussed. In addition, the method is used to analyze and compare two generic configurations of two-stage PTRs including the effect of cooling load at the mid-stage.

MATHEMATICAL FORMULATION

FIGURE 1 shows a generic configuration of two-stage PTRs assuming conductive cooling at the midstage using two separate compressors. This generic model is convenient for thermodynamic analysis and optimization and shows the important characteristics of two-stage PTRs. Exergy comes into the system from the compressors and is destroyed as it moves into the system. The product exergy is delivered to the cold reservoirs at the mid-stage and the second stage. If there is no cooling load at the midstage, the auxiliary cooler only supports the thermal losses from the first regenerator. We assume that the pressure and mass flow rate at any location at the inlet and exit of each component are given by

$$\dot{M}_j = \dot{m}_j Cos(\omega t + \phi_j) \tag{1}$$

$$P_j = P_a + p_j Cos(\omega t + \theta_j) \tag{2}$$

where ϕ_j and θ_j are the phase angle for mass flow and pressure at any inlet or exit of any component in the system, respectively. The pressure component of exergy at any location can be written as [18]

$$<\dot{E}_j>_p = \frac{RT_o}{\tau}\int_o^\tau \dot{M}_j \ln(\frac{P_j}{P_a})dt \tag{3}$$

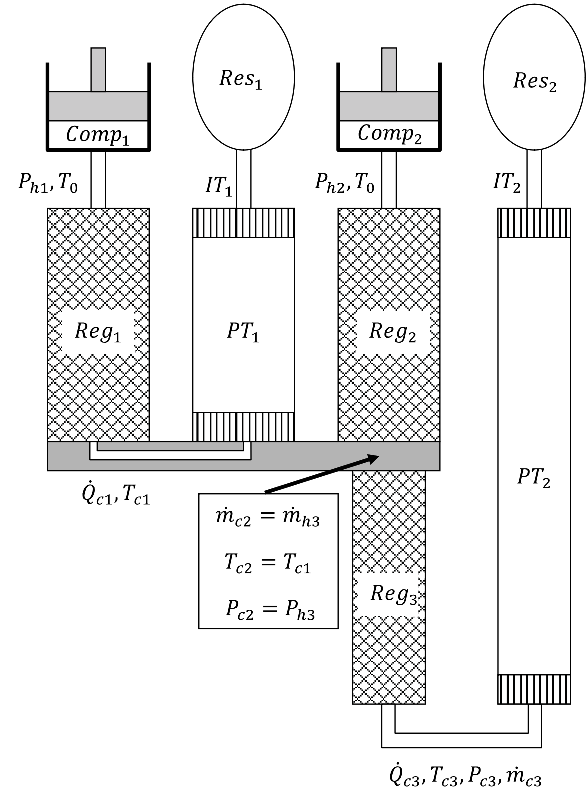

FIGURE 1. Schematic of the two-stage PTR and the parameters used in the model

In most practical PTR applications, where the amplitude ratio is relatively small (less than 0.3), Equation (3) can be simplified and the pressure component of exergy can be written as

$$< \dot{E}_j >_p = (1/2) T R_0 \, \dot{m}_j \frac{p_j}{p_a} \, Cos(\phi_j - \theta_j) \tag{4}$$

The thermal exergy transfer at the cold side of the nth regenerator, due to conduction heat transfer and ineffectiveness of the regenerator, can be written as

$$< \dot{E}_n >_{th} = (\dot{Q}_{reg} + \dot{Q}_{condn})(1 - \frac{T_o}{T_{cn}}) \tag{5}$$

where $\dot{Q}_{regn}$ can be estimated using the ineffectiveness of the nth regenerator defined by [18],

$$\lambda_n = \frac{\dot{Q}_{regn}}{\Delta H_n} \approx \frac{\dot{Q}_{regn}}{(1/\pi) C_p (T_{hn} \, \dot{m}_{hn} - T_{cn} \, \dot{m}_{cn})} \tag{6}$$

where the subscripts *hn* and *cn* denote the hot side and cold side of the nth regenerator and λ_n is its ineffectiveness. The denominator of Equation (6) represents the maximum rate of enthalpy transfer into the regenerator. Conduction heat transfer to the cold heat exchanger can be estimated assuming a linear temperature profile in the regenerator,

$$\dot{Q}_{condn} = (KA/L)_n (T_{hn} - T_{cn}) \tag{7}$$

where $(KA/L)_n$ is the effective thermal conductance of the nth regenerator and its shell. It should be pointed out that the thermal exergy transfer at the cold side of the regenerator is opposite to the direction of energy transfer at the point. Using Equations (5) to (7) the total exergy at the cold side of nth regenerator can be written as

$$\begin{aligned} <\dot{E}_{cn}> &= <\dot{E}_{cn}>_p + <\dot{E}_{cn}>_{th} \approx (1/2)RT_o\,\dot{m}_{cn}\frac{p_{cn}}{P_a}Cos(\phi_{cn} - \theta_{cn}) \\ &+ \left[(1/\pi)\lambda_n C_p (T_{hn}\,\dot{m}_{hn} - T_{cn}\,\dot{m}_{cn}) + (KA/L)_n (T_{hn} - T_{cn})\right](1 - \frac{T_0}{T_{cn}}) \end{aligned} \tag{8}$$

Oscillating flow of heat and mass transfer in regenerators is very complicated. For thermodynamic analysis we consider a simple model for the regenerator to find the bounds for cooling capacity and efficiency of two-stage PTRs. It is assumed that the mass flow rate at the cold side of the regenerator can be obtained using a linear relation based on a given, appropriately average flow conductance.

$$\dot{m}_{n+1}Cos(\omega t + \phi_{n+1}) = C_n\left[p_n Cos(\omega t + \theta_n) - p_{n+1}Cos(\omega t + \theta_{n+1})\right] \tag{9}$$

where C_n is the average flow conductance in the n^{th} regenerator. In addition, a simple model for the conservation of mass in the regenerator can be written as,

$$\dot{m}_n Cos(\omega t + \phi_n) = \dot{m}_{n+1}Cos(\omega t + \phi_{n+1}) - V_n\,\omega\left[p_n Sin(\omega t + \theta_n) + p_{n+1}Sin(\omega t + \theta_{n+1})\right] \tag{10}$$

where V_n is the properly averaged effective regenerator void volume including the influence of temperature distribution in the n^{th} regenerator. Expanding Equations (9) and (10) and applying trigonometric identities gives a set of equations that uniquely determine the relations between the phase shifts and amplitudes of pressure and mass flow rates at the cold and hot sides of the regenerator.

Assuming a phase-shift control mechanism exists to control the phase shift between the mass flow and pressure at the cold side of pulse tubes, one can optimize the exergy delivered to the reservoirs at the mid-stage and the cold stage of the two-stage PTR. The irreversibility in the pulse tube can be presented by defining its exergetic efficiency as the ratio of exergy transfer at the hot side of the pulse tube to the cold side of the pulse tube. Referring to FIGURE 1 and using exergy balance for the cold heat exchanger at the mid-stage of the two-stage PTR gives,

$$<\dot{E}_{c1}> = \dot{Q}_{c1}(\frac{T_o}{T_{c1}} - 1) + \frac{1}{\eta_{PT1}}\begin{bmatrix}\dot{Q}_{c1} + (1/\pi)\lambda_1 C_p (T_o\,\dot{m}_{h1} - T_{c1}\,\dot{m}_{c1}) + (KA/L)_1 (T_o - T_{c1}) \\ + (1/\pi)\lambda_2 C_p (T_o\,\dot{m}_{h2} - T_{c1}\,\dot{m}_{c2}) + (KA/L)_2 (T_o - T_{c2})\end{bmatrix} \tag{11}$$

where subscripts 1 and 2 correspond to regenerators 1 and 2, respectively. $\dot{Q}_{c1}$ and T_{c1} are the cooling load and temperature of the reservoir at the midstage, respectively. The bracket in the above equation represents the enthalpy flow in the pulse tube of the auxiliary cooler and η_{PT1} represents its exergetic efficiency. Equation (8) applied to the first regenerator gives the left hand side of Equation (11). Assuming the temperatures of the cold sides of the first and second regenerators are the same and equal to the temperature of the reservoir of intermediate cooling load, a relation for the cooling load at the mid-stage can be written as,

$$\dot{Q}_{c1} = \frac{(1/2)RT_o\,\dot{m}_{c1}\,\dfrac{p_{c1}}{P_a}Cos(\phi_{c1}-\theta_{c1})}{\dfrac{T_o}{T_{c1}} - 1 + \dfrac{1}{\eta_{PT1}}} - (1/\pi)\lambda_2 C_p(T_o\,\dot{m}_{h2} - T_{c1}\,\dot{m}_{c2}) \qquad (12)$$
$$-(KA/L)_2(T_o - T_{c1} - (1/\pi)\lambda_1 C_p(T_o\,\dot{m}_{h1} - T_{c1}\,\dot{m}_{c1}) - (KA/L)_1(T_o - T_{c1})$$

where T_{c1} is the reservoir temperature of intermediate cooling load. Similarly, application of equation (8) to the cold side of the third regenerator in the second stage of the cooler and the balance of exergy at the cold heat exchanger of the second stage gives a relation for the cooling load at the second stage,

$$\dot{Q}_{c2} = \frac{(1/2)RT_o\,\dot{m}_{c3}\,\dfrac{p_{c3}}{P_a}Cos(\phi_{c3}-\theta_{c3})}{\dfrac{T_o}{T_{c3}} - 1 + \dfrac{1}{\eta_{PT2}}} - (1/\pi)\lambda_3 C_p(T_{c1}\,\dot{m}_{c2} - T_{c3}\,\dot{m}_{c3}) - (KA/L)_3(T_{c1} - T_{c3}) \qquad (13)$$

Expansion of Equations (9) and (10) for the three regenerators gives 12 equations relating the amplitude of mass flow rates, amplitude of the pressure, and the phase angles at the inlet and exit of each regenerator. For a given phase shift at the cold sides of pulse tubes, assumed to be controlled by the phase shifters, and the amplitude of pressure at the driver sides as the input variables, the twelve equations can be used to find the amplitudes of pressure and flow rates and the phase shifts at all other locations. Equation (4) can be used to find the power input to the compressors,

$$\dot{W}_{comp1} = (1/2)RT_o\,\dot{m}_{h1}\frac{p_{h1}}{P_a}Cos(\phi_{h1}-\theta_{h1}) \qquad (14)$$

$$\dot{W}_{comp2} = (1/2)RT_o\,\dot{m}_{h2}\frac{p_{h2}}{P_a}Cos(\phi_{h2}-\theta_{h2}) \qquad (15)$$

Using Equations (12) to (15), the performance of the two stage pulse tube used in this study can be evaluated. The exergetic efficiency of the two stage pulse tube can be calculated

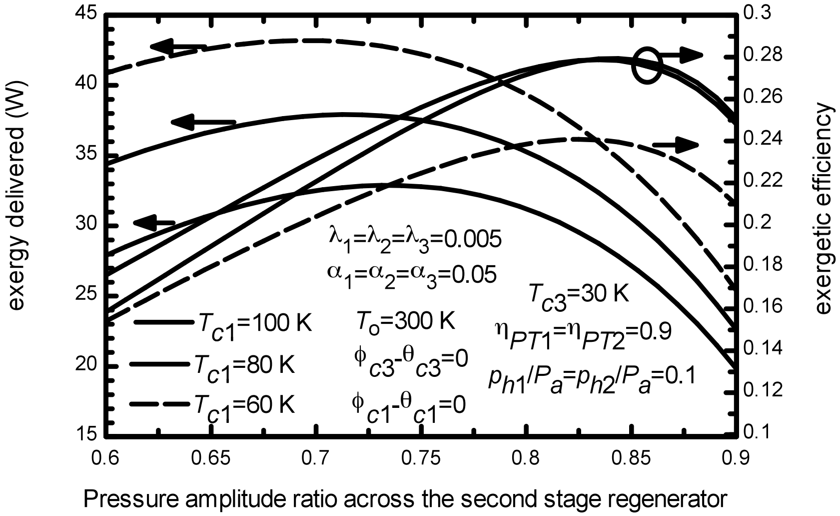

FIGURE 2. Exergy delivered at the second stage and exergetic efficiency as a function of the pressure amplitude ratio across the second stage regenerator

from equations (12) to (15) and is defined by the exergy delivered to the cold reservoir divided by the total compressor power,

$$\eta_{ex} = \frac{\dot{Q}_{c1}(T_o/T_{c1}-1)+\dot{Q}_{c2}(T_o/T_{c3}-1)}{\dot{W}_{comp1}+\dot{W}_{comp2}} \tag{16}$$

RESULTS AND DISCUSSIONS

Equations (12) to (16) give the basic equations for the calculations of the performance of the two-stage PTR used in this study. The mass flow rates and phase shifts used in these equations are obtained from the trigonometric relations given in equations (14) and (15). These relations are considerably simplified for the case of no void in the regenerator. In this case the amplitude of mass flow rates at the cold and hot sides of each regenerator are the same. The assumption of no void in the regenerators provides a thermodynamic bound for the performance of the two-stage PTR.

There are several parameters that can be controlled or used as variables to perform parametric studies and optimization of the two-stage PTR used in this study. Important dimensionless parameters that results from equations (9) and (10) are $\alpha_n = V_n\omega/C_n$, $\beta_n = \dot{m}_{cn}/C_n p_{hn}$, $mr_n = \dot{m}_{cn}/\dot{m}_{hn}$, and $\Pr_n = p_{cn}/p_{hn}$. FIGURE 2 shows the exergy delivered to the second stage reservoir defined by $\dot{Q}_{c2}(T_o/T_{c03}-1)$ and the exergetic efficiency of the two-stage PTR as a function of the pressure amplitude ratio across the regenerator of the second stage. In this calculation no load is applied to the mid-stage ($\dot{Q}_{c1}=0$). In the calculation, it is assumed that the mass flow and pressure are in-phase at the cold sides of both pulse tubes and the values of ineffectiveness for the regenerators are fixed. Important values of parameters used in the calculation are given in the figure. FIGURE 3 shows the cooling capacity and the exergetic efficiency as a function of the cold end temperature of the second stage for fixed values of pressure ratio across

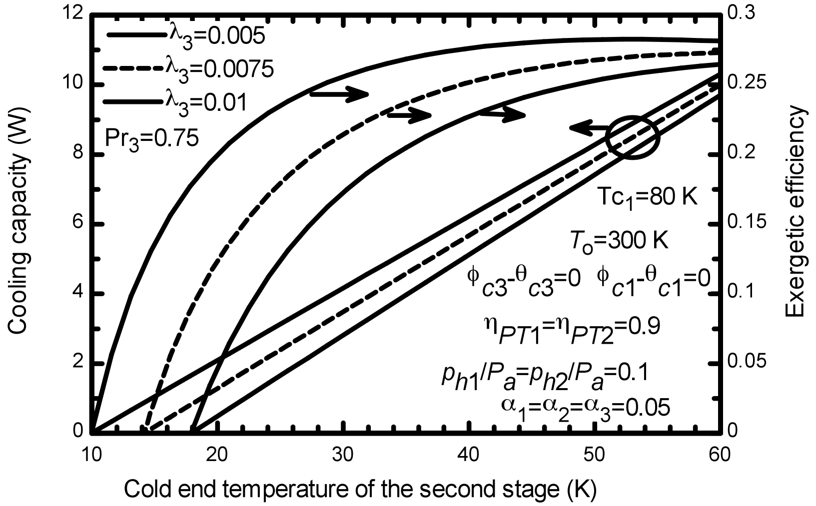

FIGURE 3. Cooling capacity at the second stage and exergetic efficiency as a function of the cold end temperature of the second stage.

regenerator 3 and the mid-stage temperature. The results are given for three values of second stage regenerator ineffectiveness. The cooling capacity is approximately a linear function of the cold reservoir temperature as typically occurs in practice. The rate of increase in the exergetic efficiency decreases as the second stage temperature approaches the mid-stage temperature. FIGURE 4 shows the cooling capacity and efficiency diagram corresponding to FIGURE 2 when the pressure ratio across regenerator 3 is used as a parameter. In this calculation the ineffectiveness of regenerator 3 is used as a parameter to show its effect on the performance of the two-stage PTR. Other parameters are given in the figure. The loop-shaped curves in the figure indicate a compromise between the cooling capacity and efficiency of the two-stage PTR for the case studied.

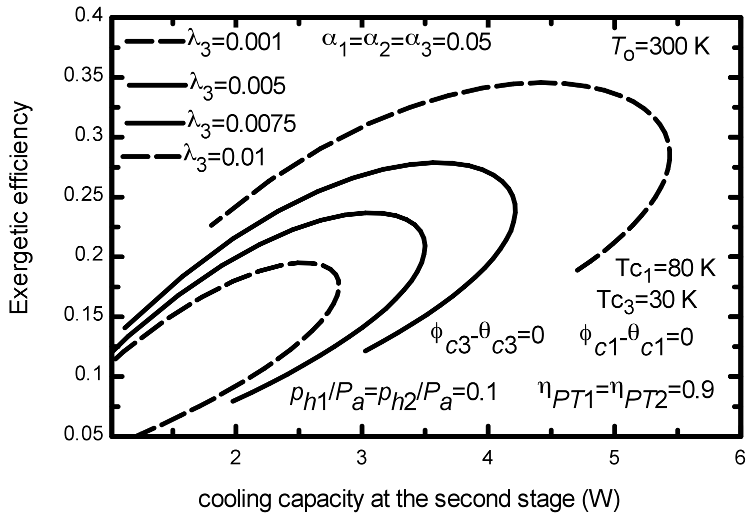

FIGURE 4. Cooling capacity and efficiency diagram for different values of ineffectiveness of the second stage regenerator.

CONCLUSIONS

A control thermodynamic model for a two-stage PTR is formulated using the exergy analysis of the refrigerator. A simple model for the regenerators was developed based on given flow conductance. An important dimensionless parameter for parametric study and optimization of the refrigerator was developed. A cooling capacity and efficiency diagram for the refrigerator is presented and the effect of the regenerator ineffectiveness and mid-stage temperature on cooling capacity and efficiency is evaluated. It is shown that depending on the constraints imposed on the system, a compromise between cooling capacity and efficiency is possible. Application of the method to a more conventional two-stage PTR is under investigation.

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